Acoustics

Mathematics to make your ears ring

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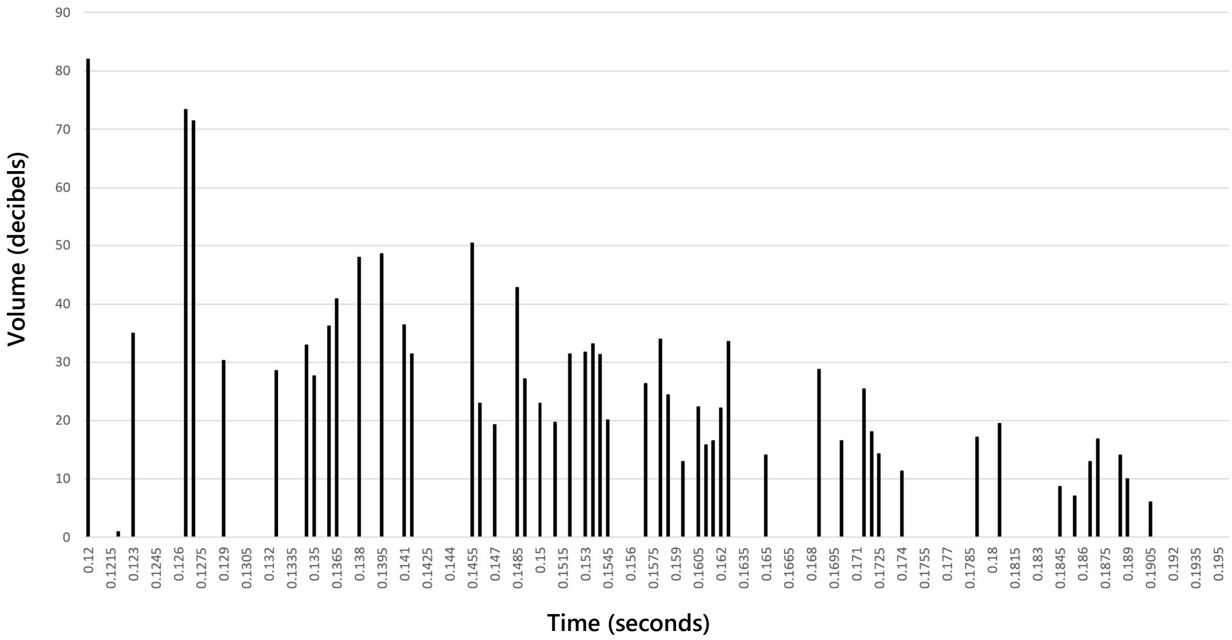
Measuring echoes

Going to the Sydney Opera House and watching a live performance, perhaps from an opera or orchestra, is a very special experience. It does not matter how many times you may hear a piece of music through your headphones or at home in your living room; listening to that same song being played in your presence sounds fundamentally different. Why is this the case?

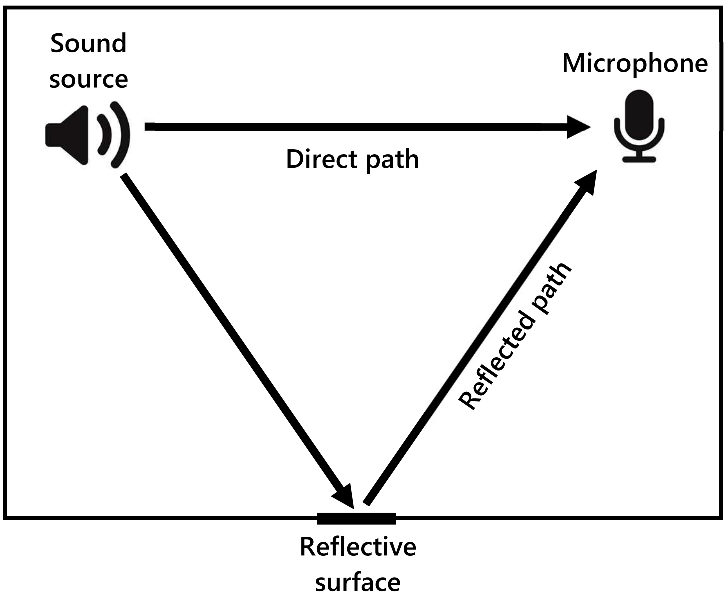
There are many contributing factors, but one of the most significant is the space that you are in when you listen. The sheer volume of the Sydney Opera House theatres has a profound effect on what music sounds like when you hear it there. The theatres of the are specifically designed with acoustic characteristics that have been carefully fine-tuned to affect the sonic experience we have when we enter them.

One of the methods used by acoustic engineers is to use sensitive microphones and take recordings to analyse the effect of the space on the experience of listeners. One tool used to analyse these recordings is called a *reflectogram*, and a simplified example of one can be seen on the next page.



Reflectograms measure the amplitude, or volume, of sound over time. The vertical axis shows the volume (measured in decibels) of the recorded sound, while the horizontal axis shows the time (measured in seconds) for that sound to be picked up by the microphone.

In the reflectogram above, a single loud sound has been played in the space, which is represented by the vertical bar at the left-most part of the graph. The other vertical bars represent echoes of that original sound which reflect off various surfaces within the space before arriving at the microphone. The diagram below shows one way this could occur.



Since the sound must travel further along the *reflected path* compared to the *direct path*, it takes longer to arrive at the microphone and is also lower in volume when it arrives. This is the reason that the vertical bars tend to decrease in height as we move further to the right in the reflectogram.

Sound travels at varying speeds according to factors like temperature, humidity and air pressure. If we take the speed of sound in a particular space to be 331 metres per second, then answer the following questions:

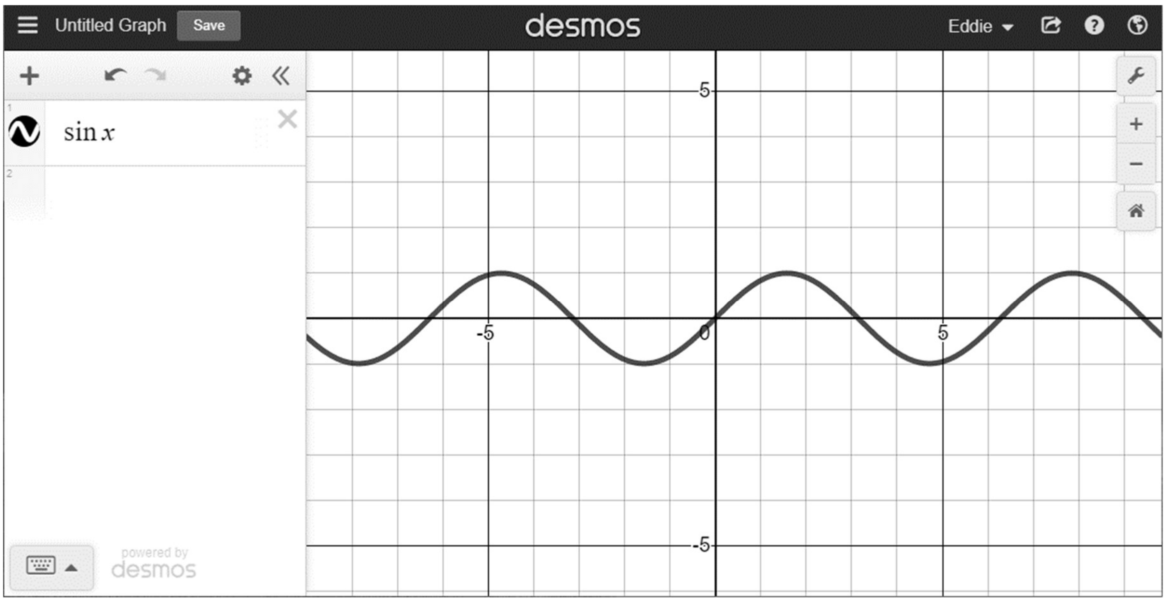
a) How long would it take for a sound to travel 100 metres? Provide your answer in seconds, accurate to 1 decimal place.
b) The original sound in the reflectogram arrives after 0.12 seconds. How far must the sound source be from the microphone? Provide your answer in metres, accurate to 1 decimal place.
c) There are noticeable spikes in the reflectogram after 0.126 seconds and 0.1455 seconds. How far must the sound have travelled to have produced these echoes? Provide your answer in metres, accurate to 1 decimal place.

Melody and harmony

This next activity makes use of the *Desmos graphing calculator* to dynamically explore the nature of musical notes. You can access it here: http://www.desmos.com/calculator

Most of us enjoy music in some form, but we seldom think about why some musical notes sound harmonious together while others do not. To understand this part of musical theory, we need to recognise that sound is a vibration that travels through air, water, and other materials. Musical instruments make sound by vibrating the air within and around them in predictable ways.

To see a visualisation of this, use an internet browser to open the Desmos graphing calculator. On the left-hand menu, type in "sin x" (no space is required between sin and x, but Desmos will add this in automatically). If you did it correctly, the following graph should appear:

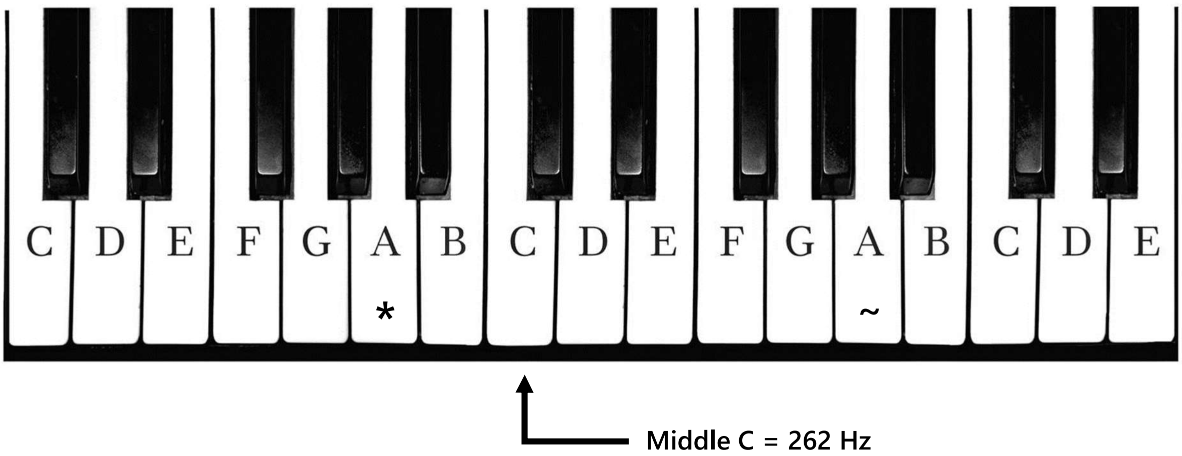


What you are seeing is a visual representation of a mathematical function called *sine* (“sin” is just an abbreviation, but it is still pronounced so it rhymes with “wine”). Its wavy geometry makes it perfectly suited to representing all kinds of natural phenomena that involve oscillation (such as electromagnetic radiation or rising and falling tides).

The high sections of the sine wave are called *crests*, while the low sections are called *troughs*. If we take this sine wave to represent a simplified version of a musical note, then crests represent regions of the air at high pressure, while troughs represent regions at low pressure. The interaction between these crests and troughs is what our ears and brains perceive as sound.

When the crests and troughs are bunched up more closely together, this represents a musical note at a higher pitch. Since the crests and troughs occur more frequently, we call this a higher *frequency*. Conversely, when the sound wave is at a lower frequency, this leads to a musical note at a lower pitch.

Every musical note has its own specific frequency. For instance, on a piano, middle C has a frequency of approximately 262 Hertz – this means that its sound wave would have 262 crests (and therefore also 262 troughs) in the space of a single second.



Knowing about frequency is the key to understanding why some notes are harmonious while others are discordant. For instance, on the piano keyboard, the A note to the left of middle C (marked with an asterisk *) has a frequency of 220 Hz. Remember that it is lower because the notes become lower in pitch as we move to the left of the keyboard. If we were to create a note with double the frequency – 440 Hz – it would match nicely with this A note. That is because the two notes represent an octave: the higher note is the A note on the right of middle C (marked with a tilde ~).

We can see the mathematical relationship between these two notes back in Desmos. On the line underneath “sin x”, type in “sin $2x$”. You will notice a second graph appear atop the first one. Do you notice how every second wave of sin $2x$ lines up with each full wave of sin x? This mathematical phenomenon is what we experience sonically as harmony.

The octave is not the only kind of harmony that we have heard before. Another common harmony is called the *perfect fifth*. The frequency of two notes in this harmony is in the ratio 2:3 – another way of saying this is that one note has a 50% higher frequency than the other. We can represent this visually by leaving the sin $2x$ graph alone but modifying the sin x equation so that it reads sin $3x$. You can see that every second wave of sin $2x$ matches up with every third wave of sin $3x$, which again is a harmonious relationship.

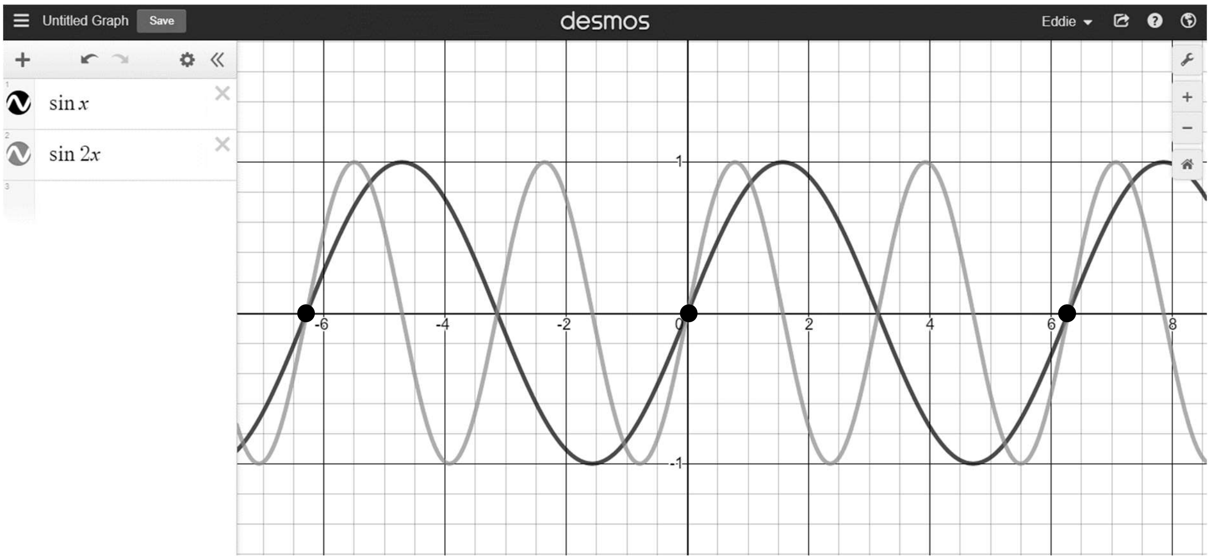
Manipulating these sine waves also allows us to see why some notes sound discordant. If you have ever tried playing two adjacent notes on a piano keyboard, you will notice that they sound very unpleasant together. This is because the ratio between their frequencies is 15:16. You can visualise this by graphing sin $2x$ along with sin $1.875x$. The two graphs almost never line up perfectly (you will need to scroll further to the right or left in Desmos to see the first place that this actually occurs).

Solutions

a) Time = 0.3 seconds
b) Distance = 39.7m
c) First echo travels 41.7m, second echo travels 48.2 metres

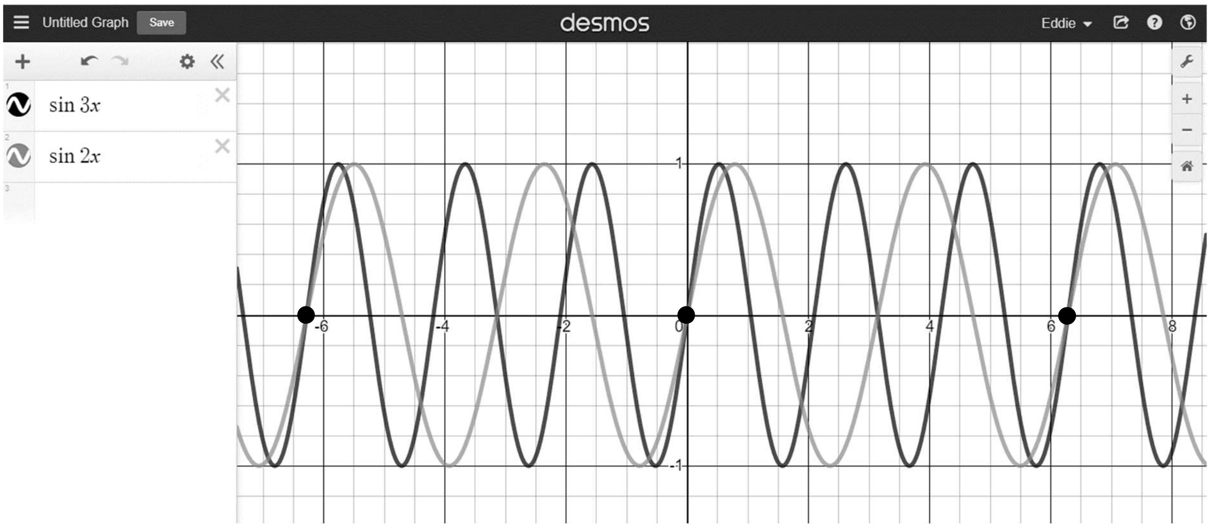
Graphs of sin x and sin $2x$ (representing an octave)

Note: the black circles below show places where the two graphs line up with each other. They have been added manually here and do not appear on the original Desmos graph.



Graphs of sin $3x$ and sin $2x$ (representing an octave)

Note: as above, black circles have been added manually here for illustrative purposes.



Graphs of sin $1.875x$ and sin $2x$ (representing a semitone)