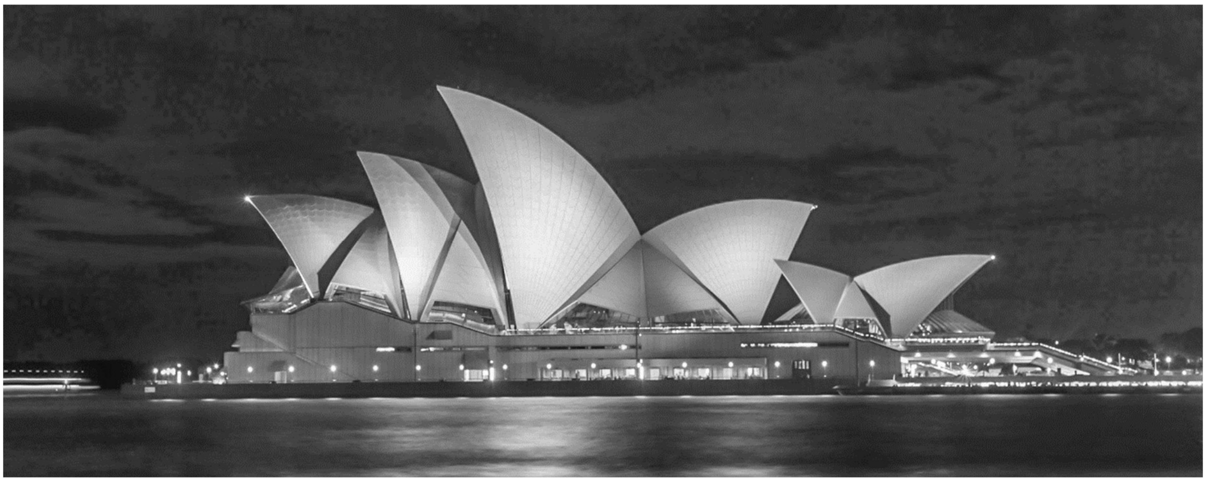
Nature Inspiring Design



Photograph by Jerry Skinner, CC BY-SA

Organic design

Architectural forms vary around the world and through time. Different cultures throughout history have designed their buildings around several factors, including: their specific aesthetic values, the raw materials they had access to and the engineering techniques they had developed.

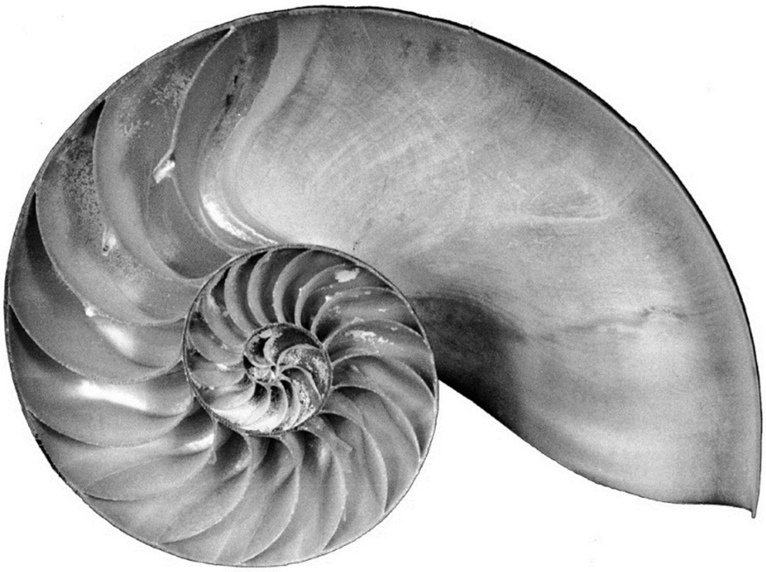
Utzon's mission, when designing the Sydney Opera House, was to conceive of a structure that would have global appeal. From the first moment that it was proposed, it was intended to be an international attraction – both for theatregoers and the performers they were coming to see. One of the ways that he ensured his building would be appreciated and marvelled at by people from all over the world was to employ a universal design language, namely that of the natural world.

Curved shells

City buildings such as skyscrapers are dominated by straight lines, and with good reason. They lend predictability, strength and uniformity to a building's design as well as its construction. This is one of the many reasons why the elegant curves of the Sydney Opera House stand out on the Sydney skyline.

While the brilliantly reflective external shapes of the Sydney Opera House are known as sails, the internal concrete structures that give the building its strength and integrity are more commonly referred to as shells. This is fitting because one of the organic shapes that Utzon alludes to in his design is that of a seashell, a perfect comparison to make given the maritime setting of the Sydney Opera House.

Seashells are curved in hundreds of different ways, dependent on the kind of creature that fashioned them. One of the most impressive is that of the nautilus, whose shell can be seen in cross-section on the following page:

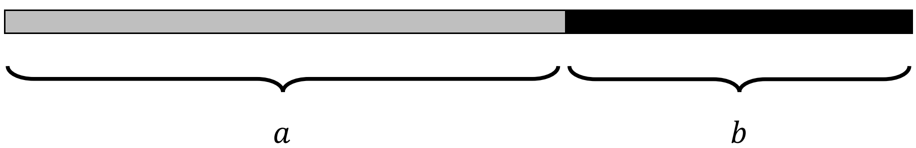


Photograph by Lisa Ann Yount, public domain

This shell's characteristic spiral is governed by a simple number, called the *golden ratio*. This ratio, which is roughly equal to the value 1.618, arises when we divide an object (such as a line, area, volume or angle) into two parts such that:

The larger part divided by the smaller part	is equal to	*The whole object divided by the larger part*

You can see an example in the diagram below.



Use a ruler to accurately measure the total width of this shape, then measure the width of the grey (a) and black (b) sections. Fill in these values into the table below:

Grey section (a)	Black section (b)	Total width ($a+b$)

If you've measured precisely, you should be able to demonstrate the following relationship:

$$\frac{a}{b} = \frac{a+b}{a} \approx 1.618 \ldots$$

(Don't worry if your measurements and calculations are slightly off the expected value – we are always limited by the accuracy of our measuring instruments, and standard rulers are not designed for rigorous scientific precision!)

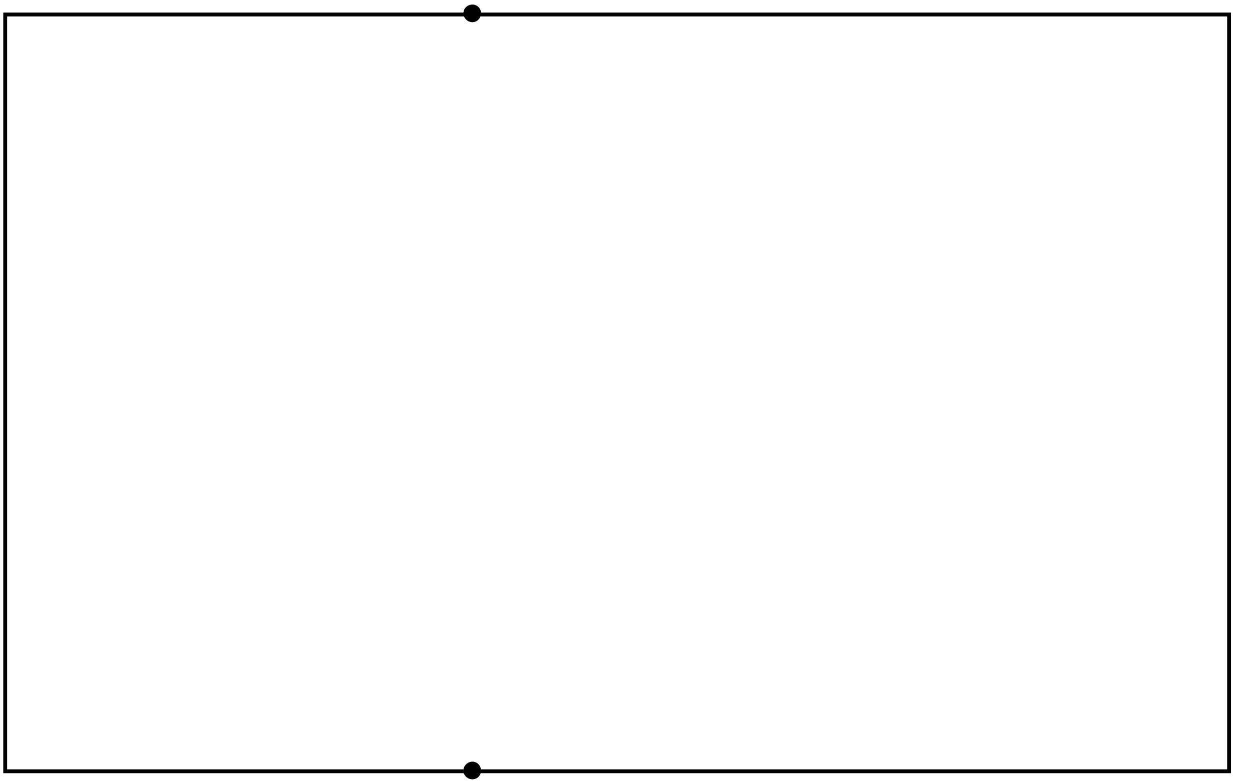
The golden ratio is often denoted by the Greek letter ϕ (pronounced "phi"). If we use this notation, noting that $\phi = \frac{a}{b}$, we can rewrite the equation above in the following fashion:

$\phi = \dfrac{a+b}{a}$	by definition
$\phi = \dfrac{a}{a} + \dfrac{b}{a}$	splitting up the right-hand fraction into two fractions
$\phi = 1 + \dfrac{1}{\phi}$	since $\frac{a}{a} = 1$ and $\frac{b}{a}$ is the reciprocal of $\frac{a}{b}$
$\phi^2 = \phi + 1$	multiplying both sides through by ϕ
$\phi^2 - \phi - 1 = 0$	rearranging all terms onto the left-hand side

What you see on the final line above is a *quadratic equation* that can be solved to calculate the golden ratio exactly (so that we don't need to worry about imprecise rulers to try and find its value!).

If you've encountered and learned about quadratic equations already, try solving the equation above by *completing the square* or using the *quadratic formula*. The answer will involve irrational numbers (surds), which you can then evaluate with a calculator to see the golden ratio's exact value!

We can see the relationship between ϕ and the nautilus shell by seeing how it interacts with the shape below.



This shape is known as a *golden rectangle*, as its side lengths are formed in the golden ratio. To make things a little easier to describe, let's name this shape *Rectangle 1.* Comparing this to the black and grey diagram we saw previously, Rectangle 1's width would be a and Rectangle 1's height would be b. You can measure and confirm that $\frac{a}{b}$ is still equal to ϕ.

Rectangle 1 width (a)	Rectangle 1 height (b)	Ratio $\left(\frac{a}{b}\right)$

Two points have been marked on the upper and lower sides of the rectangle. If you join them together, you will have divided the rectangle into a square (on the right) and a smaller rectangle (on the left; let's call this *Rectangle 2*). The striking fact about this, and what makes the golden rectangle special, is that *Rectangle 2 is also a golden rectangle*. Measure its width and height to confirm that the golden ratio appears again:

Rectangle 2 width (x)	Rectangle 2 height (b)	Ratio $\left(\frac{b}{x}\right)$

You can continue this division process by measuring carefully and dividing Rectangle 2 into a square (at the top) and an even smaller rectangle (*Rectangle 3*) on the bottom. If you keep doing this, forming squares in the anticlockwise direction as you go around, you'll create a spiral of squares that turn inward and converge somewhere near the middle of *Rectangle 1*. Turn to the end of this activity sheet to see what it this should look like if you repeat this step many times.

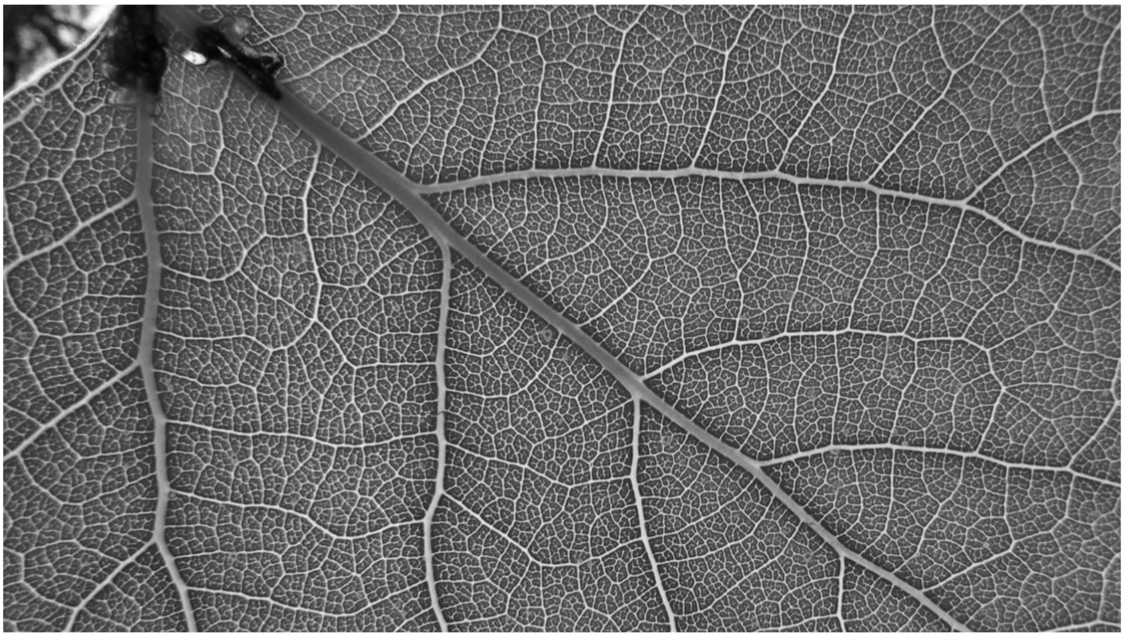
What does this have to do with the nautilus shell? To see the relationship, draw a circular arc joining the bottom-right corner and top-left corner of the largest square. Then, connect this arc to a new arc that joins the top-right corner and bottom-left corner of the next largest square. Continue the arc inward in an anti-clockwise direction – and you will have replicated the basic spiral shape of the nautilus shell.

Worlds within worlds

Another aspect of Utzon's design that reflected the natural world was something he called *additive architecture*. This was the idea that a building could be composed of components that demonstrated the characteristics of organic growth. This is clear in the design of the Sydney Opera House: each sail is made of distinct concrete ribs, each rib is made of stacked chevrons, and each chevron is composed of individual tiles.

Speaking of their organic inspiration, sitting within the Bennelong Restaurant affords a beautiful interior view of the ribs themselves, which genuinely makes it appear that you are sitting within the ribcage of a gigantic sea creature.

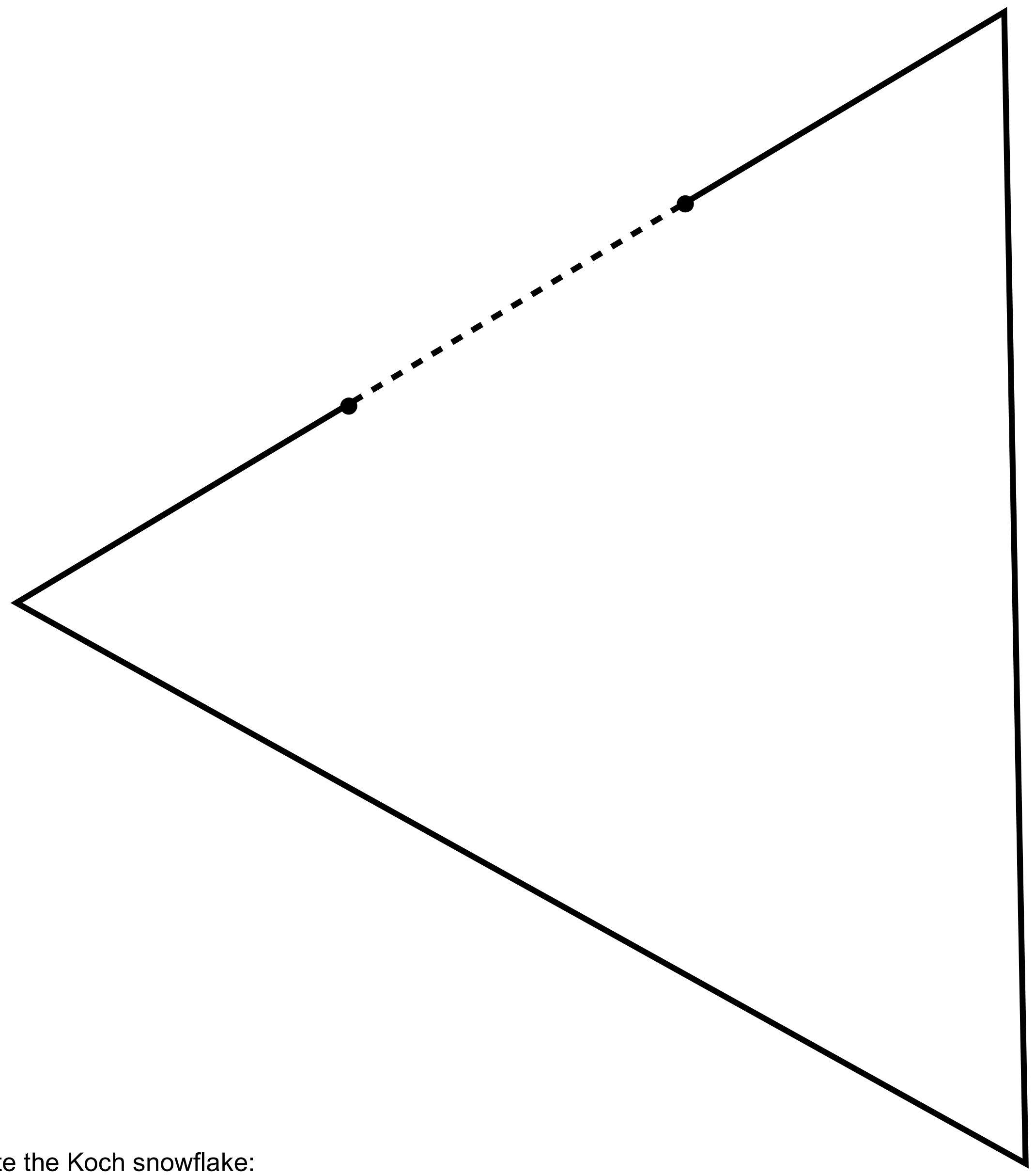
This composition is parallel to the way a tree is made of branches, each branch is made of stems, each stem bristles with leaves and even individual leaves have branch-like structures within them.



Photograph by Curran Kelleher, CC-BY-SA

Many mathematical objects exhibit these same geometric patterns. One such category of objects are the *fractals* – shapes that often have the property of being *self-similar*. This means that they have increasing levels of complexity at deeper levels of magnification, just like trees and the Sydney Opera House. One of the most famous fractals is simple enough that we can explore it together – it's called the *Koch snowflake*.

The Koch snowflake begins with an equilateral triangle, like the one on the following page. As with our exploration of the golden rectangle, we'll name objects to make it easier to keep track of things. Let's call this first shape the "A Triangle".

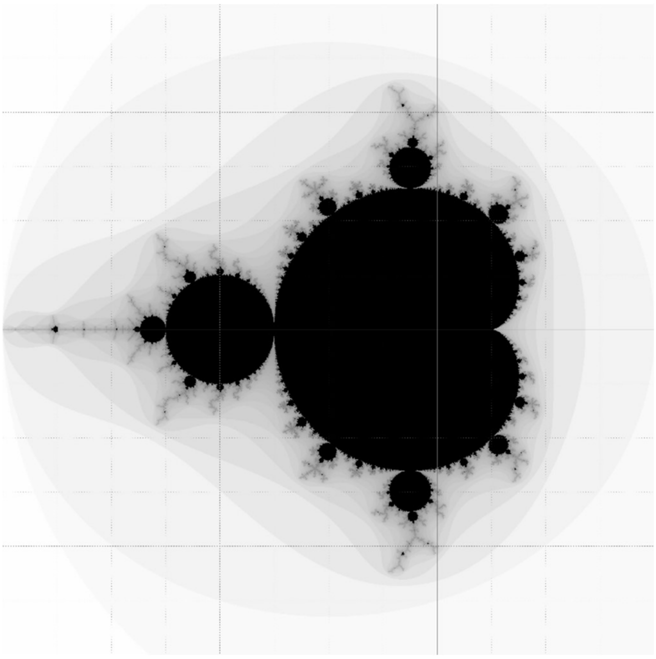


To create the Koch snowflake:

- Divide each side of the A Triangle into thirds (I've helped you with the first side – use a ruler to do the same with the other two sides).
- Use the middle third of each length (which I've marked on the first side as a dotted line) as the *base of a new equilateral triangle* that protrudes directly out from the original triangle. In this way, you'll draw three new smaller triangles (which we can call B Triangles) that look like spikes growing out from the original triangle.
- Repeat this process with *every outward-facing side* of the new figure. This means you'll have to draw *twelve new triangles* that are even smaller than before (these will be called C Triangles). Six of the C Triangles will be on the leftover sections of the A Triangle; the other six will be on the outward-facing sides of the B Triangles.
- Keep on going as far as you can go (and have the time to draw)!

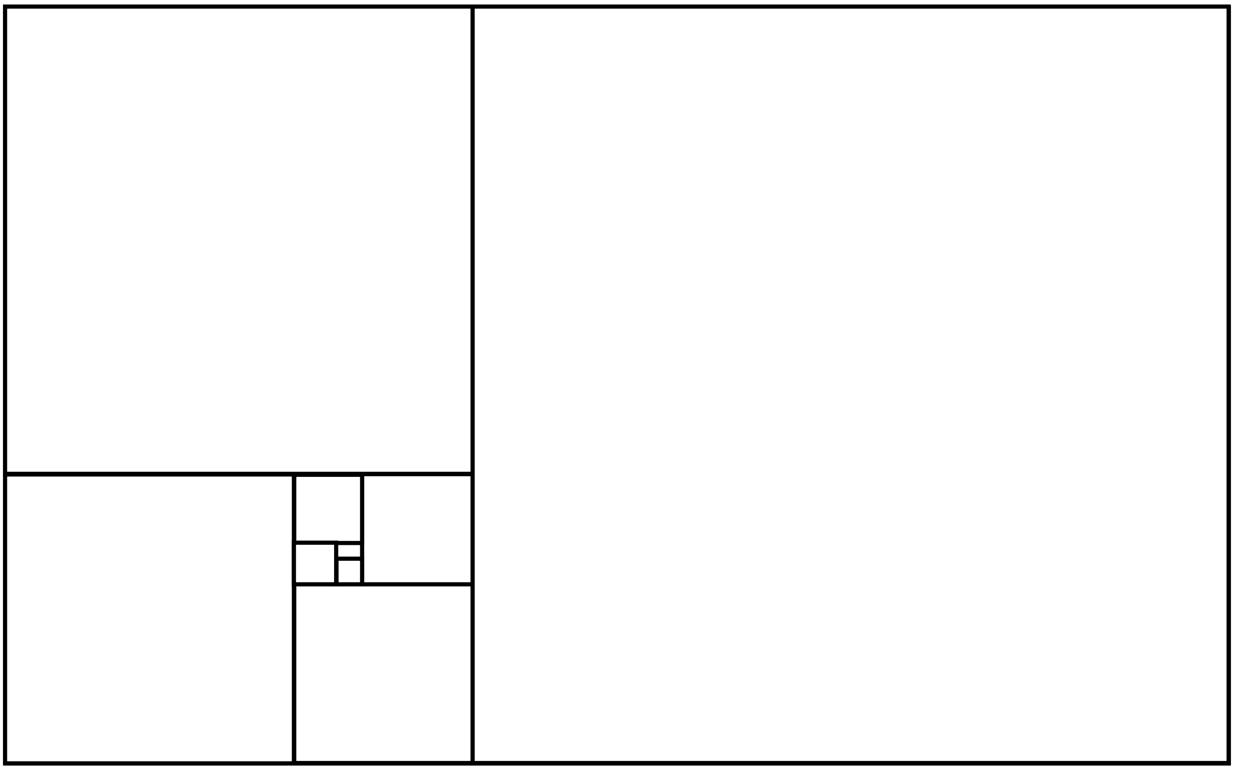
Turn to the end of this activity sheet to see what it this should look like if you repeat this step many times.

Most fractals that are explored by mathematicians require computers and sophisticated programs to draw them, due to their infinite levels of complexity. The strange and bizarre *Mandelbrot* set (shown below), named after mathematician Benoit Mandelbrot, is a perfect example of this. However, if you're interested in exploring further shapes like these that can be drawn and explored by hand, search for information about the *Sierpinski triangle* and the *Pythagoras tree*.



Computer render by Lars Rohwedder, CC-BY-SA

Golden rectangle divided into squares



Koch snowflake